Hong Kong Mathematics Olympiad (2024/25)
Heats – Group Event
香港数学竞赛 (2024/25)
初赛团体项目

INSTRUCTIONS

1. Before the start of the examination, you should follow the announcement to first write your School ID, School name, seat Number and your name, in the appropriate space provided on the answer sheet.
   宣布开考前，学生须遵照司仪的指示在答题纸适当位置填写你的学校编号、学校名称、座位编号及你的姓名。

2. This paper consists of **TWO** sections, A and B.
   本试卷分**两**部分，即甲部和乙部。

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in the attached Answer Sheet. Do not write in the margins. Answers written in the margins will not be marked.
   本试卷各题均须作答，答案须写在随附的答题纸中预留的空位内。不可在边界以外位置书写。写于边界以外的答案，将不予评阅。

4. Unless otherwise stated, all answers should be given in exact numerals in their simplest form.
   除特别指明外，所有答案须以数字的真确值表达并化至最简。

5. No approximation is accepted.
   不接受近似值。

6. Unless otherwise specified, all working **NEED NOT** be shown.
   除特别指明外，**不须**列出所有算式。

7. The diagrams in this paper are not necessarily drawn to scale.
   本试卷的附图不一定依比例绘成。

8. No extra time will be given to candidates for filling in your School ID, School name, seat Number and your name after the 'Time is up' announcement.
   司仪宣布停笔后，考生不会获得额外时间填写你的学校编号、学校名称、座位编号及你的姓名。

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Part A
甲部

1. Let α, β and γ be the roots of $2x^3-3x^2-4x-5=0$. Find the value of $(\alpha+1)(\beta+1)(\gamma+1)$.

   设 α、β 及 γ 为 $2x^3-3x^2-4x-5=0$ 的根。求 $(\alpha+1)(\beta+1)(\gamma+1)$ 的值。

2. Find the sum of all real solution(s) satisfying $\left(x^2-9x+19\right)^{7x^2+6x-1}-1=0$.

   求所有满足于 $\left(x^2-9x+19\right)^{7x^2+6x-1}-1=0$ 的实数解之和。

3. Let $f_1(x)=\dfrac{x}{1-x}$ and $f_n(x)=f_{n-1}\left(f_1(x)\right)$, where $n=2, 3, 4, \ldots$. Find the value of $f_{2025}(2)$.

   设 $f_1(x)=\dfrac{x}{1-x}$，及 $f_n(x)=f_{n-1}\left(f_1(x)\right)$，其中 $n=2, 3, 4, \ldots\ldots$。求 $f_{2025}(2)$ 的值。

4. Given that
   $A=\sin^2(1°)+\sin^2(3°)+\sin^2(5°)+\cdots+\sin^2(20240277°)+\sin^2(20240279°)$,
   $B=\cos^2(1°)+\cos^2(3°)+\cos^2(5°)+\cdots+\cos^2(20240277°)+\cos^2(20240279°)$,
   Find the value of A^2-B^2.
   已知
   $A=\sin^2(1°)+\sin^2(3°)+\sin^2(5°)+\cdots+\sin^2(20240277°)+\sin^2(20240279°)$,
   $B=\cos^2(1°)+\cos^2(3°)+\cos^2(5°)+\cdots+\cos^2(20240277°)+\cos^2(20240279°)$,
   求 A^2-B^2 的值。

5. In Figure 1 , the circle passes through two vertices of the square and touches one side of the square. If the radius of the circle is 10 , find the area of the square.
图一中，一个圆形通过一个正方形的两个顶点，且与该正方形的其中一边相切。设该圆的半径10，求该正方形的面积。

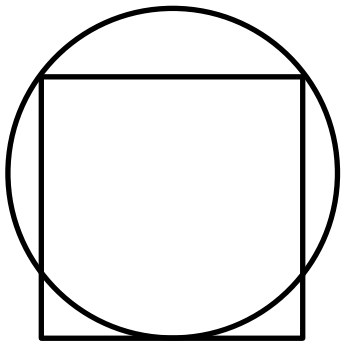

Figure 1
图一

Part B
乙部

6. Given that $f(x)=\frac{9^x}{9^x+3}$, find the value of $f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+...+f\left(\frac{2024}{2025}\right)$.
已知 $f(x)=\frac{9^x}{9^x+3}$，求 $f\left(\frac{1}{2025}\right)+f\left(\frac{2}{2025}\right)+f\left(\frac{3}{2025}\right)+...+f\left(\frac{2024}{2025}\right)$ 的值。

7. A positive integer X equals to the sum of the square of its four smallest positive factors, find the largest prime factor of X.
一个正整数 X 相等于它的四个最小正因子的平方和，求 X 的最大质因子。

8. In Figure 6, point P lies inside ΔABC such that $\angle PAB = 12°$, $\angle PBA = 18°$, $\angle PCA = 30°$ and $\angle PAC = 42°$. Find $\angle PCB$.
图 6 中，点 P 是 ΔABC 内一点使得 $\angle PAB = 12°$、$\angle PBA = 18°$、$\angle PCA = 30°$ 及 $\angle PAC = 42°$。求 $\angle PCB$。

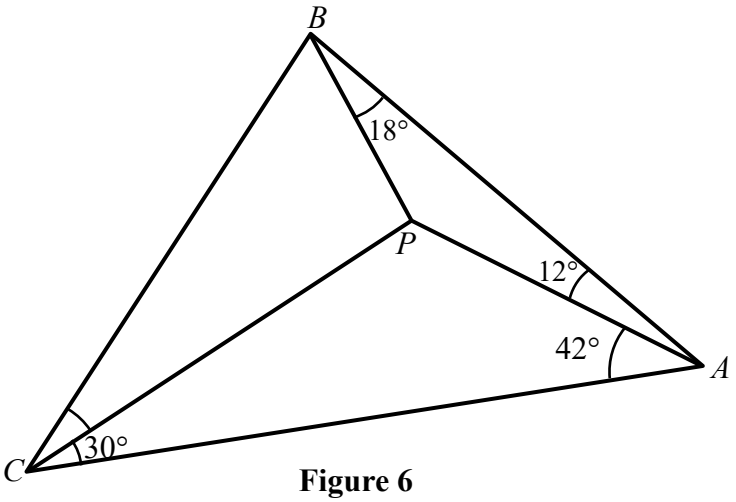

Figure 6
图 6

9. Given that $0° \le \theta \le 45°$ and $16^{\tan^2\theta} + 16 = 8\left(8^{\tan^2\theta}\right) + 2\left(2^{\tan^2\theta}\right)$, find θ.

已知 $0° \le \theta \le 45°$ 及 $16^{\tan^2\theta} + 16 = 8\left(8^{\tan^2\theta}\right) + 2\left(2^{\tan^2\theta}\right)$，求 θ。

10. In Figure 7, $ABCD$ is a square and $BCDE$ is a cyclic quadrilateral. If $AE \times CE = 28$ and $BE \times DE = 96$, find the perimeter of the square $ABCD$.
在图七中，$ABCD$ 是一个正方形及 $BCDE$ 是一个圆内接四边形。若 $AE \times CE = 28$ 及 $BE \times DE = 96$，求正方形 $ABCD$ 的周界。

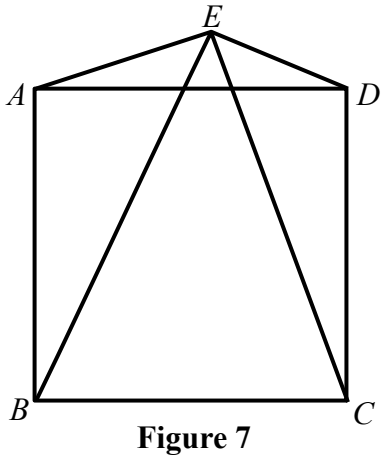

Figure 7
图七

END
完